

COMPUTING FOR SKA

**MULTIPLIER-FREE FPGA LUT
IMPLEMENTATION OF RADIO-
ASTRONOMICAL SIGNAL PROCESSING
USING DISTRIBUTED ARITHMETIC
&
MASSEY SCIENCE SPINOFFS IN SKA
RELATED Ph.D. PROJECTS**

Presented by: Rezaul Hasan, Ph.D. (UCLA), Director

**Center for Research in Analog and VLSI microsystem dEsign
(CRAVE), School of Engineering and Advanced Technology,
Massey University Auckland 0632, New Zealand**

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DISTRIBUTED ARITHMETIC

- **Well-known method to save resource in MAC structures**
- **Trades memory with combinatorial computation logic**
- **Ideal to implement DSP in LUT based FPGA**
- **Original paper by Peled and Liu (Princeton EE) in IEEE Transactions on Acoustics, speech and signal processing, “A new hardware realization of digital filters”**
- **Allows novel exploitation of the symmetry and periodicity of DFT/DCT coefficients in minimizing hardware**
- **Maybe suitable for very high speed BIG-DATA applications**

COMPUTING FOR SKA**UNDERLYING ALGORITHM OF DISTRIBUTED ARITHMETIC**

Most signal processing/ Astronomical signal processing ends-up being a collection of sum-of-products of coefficients (often with exploitable symmetry and periodicity) and signal data. For example, the output quantity, Y being given by the discrete-time digital representation over time T as:

$$Y = A_0 \times X_0 + A_1 \times X_1 + A_2 \times X_2 \dots \dots \dots + A_{T-1} \times X_{T-1}$$

Where A are coefficients and X are data-stream.

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So that we can write,

$$Y = \sum_{t=0}^{T-1} A_t \times X_t$$

Using for simplicity, sign magnitude binary representation of the data X_t

$$Y = \sum_{t=0}^{T-1} A_t \times \left[\sum_{b=0}^{N-1} X_{t,b} \times 2^b \right]$$

Where, $X_{t,b}$ is the binary value at the b^{th} bit location of the data X_t . Next we can interchange the two summations and bring them in front, so that,

$$Y = \sum_{b=0}^{N-1} \left[\sum_{t=0}^{T-1} A_t \times X_{t,b} \times 2^b \right]$$

COMPUTING FOR SKA**UNDERLYING ALGORITHM OF DISTRIBUTED ARITHMETIC**

Now, we look at the values of the sum,

$$\sum_{t=0}^{T-1} A_t \times X_{t,b} = f(b), \text{ say,}$$

For $b=0$ position,

$$f(0) = [A_0 \times X_{0,0} + A_1 \times X_{1,0} + A_2 \times X_{2,0} + \dots + A_{T-1} \times X_{T-1,0}]$$

For $b=1$ position,

$$f(1) = [A_0 \times X_{0,1} + A_1 \times X_{1,1} + A_2 \times X_{2,1} + \dots + A_{T-1} \times X_{T-1,1}]$$

For $b=2$ position,

$$f(2) = [A_0 \times X_{0,2} + A_1 \times X_{1,2} + A_2 \times X_{2,2} + \dots + A_{T-1} \times X_{T-1,2}]$$

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UNDERLYING ALGORITHM OF DISTRIBUTED ARITHMETIC

For $b=3$ position,

$$f(3) = [A_0 \times X_{0,3} + A_1 \times X_{1,3} + A_2 \times X_{2,3} + \dots + A_{T-1} \times X_{T-1,3}]$$

- .
- .
- .
- .

For $b=N-1$ position,

$$f(N-1) = [A_0 \times X_{0,N-1} + A_1 \times X_{1,N-1} + A_2 \times X_{2,N-1} + \dots + A_{T-1} \times X_{T-1,N-1}]$$

So, Y can be found by just shifting (matter of wire connection) and adding the f s.

$$Y = f(0) \times 2^0 + f(1) \times 2^1 + f(2) \times 2^2 + \dots + f(N-1) \times 2^{N-1}$$

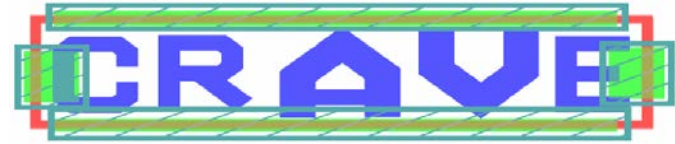
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UNDERLYING ALGORITHM OF DISTRIBUTED ARITHMETIC

But, each $f(b)$ for each bit position can have at most 2^T distinct values which can be designated as $fval(0), fval(1), fval(2), \dots, fval(2^T-1)$. These distinct values can be pre-stored in FPGA mux-based LUTs, and, selected for each $f(b)$ by using the address:

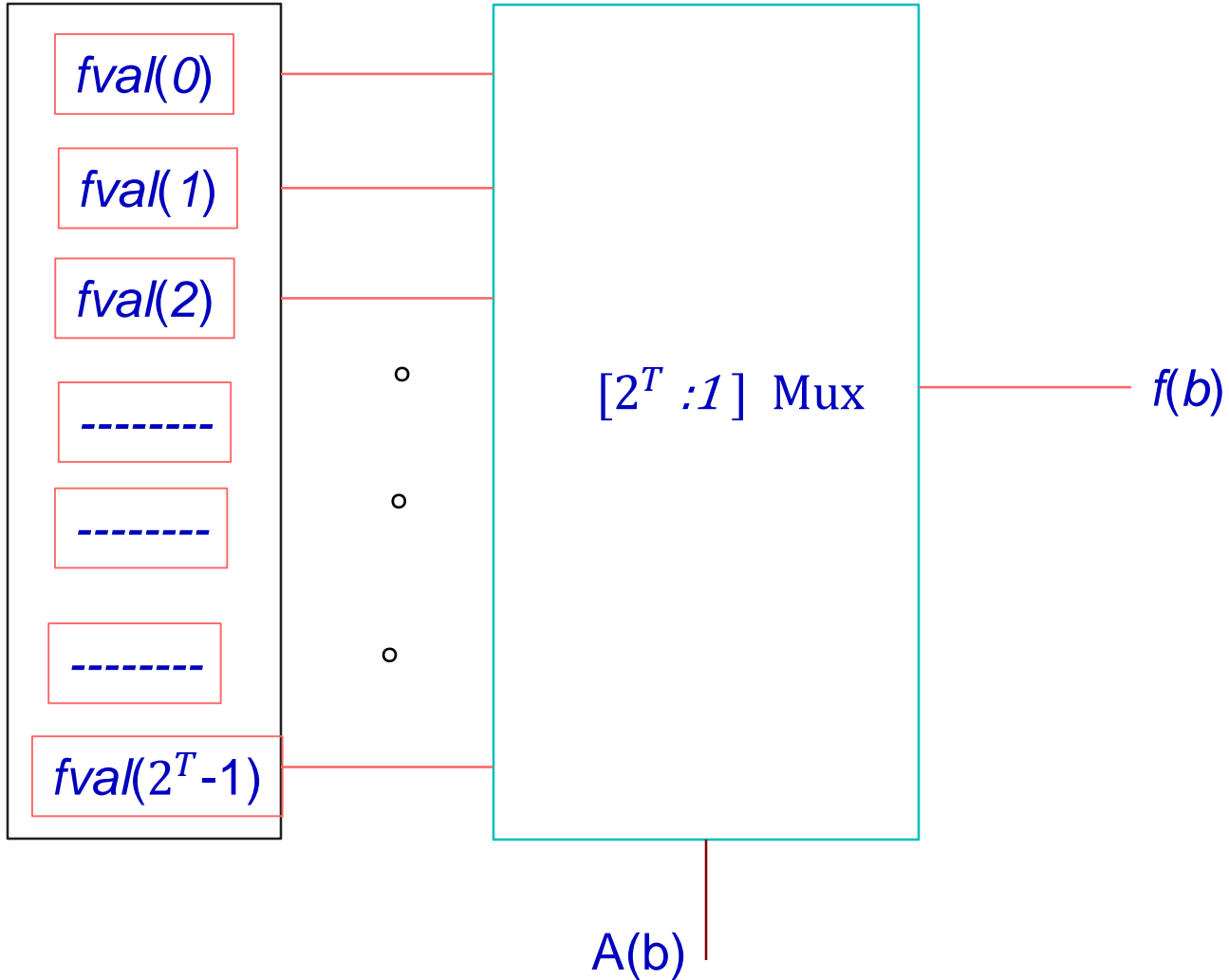
$$A(b): [X_{0,b} \ X_{1,b} \ X_{2,b} \ X_{3,b} \ \dots \ X_{T-1,b}] \text{ into a } [2^T : 1] \text{ Mux}$$

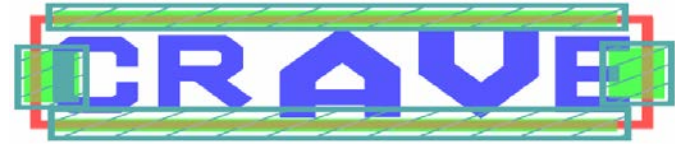
STRATIX series FPGA have lot of adders and lookup tables (memory + mux) that can implement distributed arithmetic.



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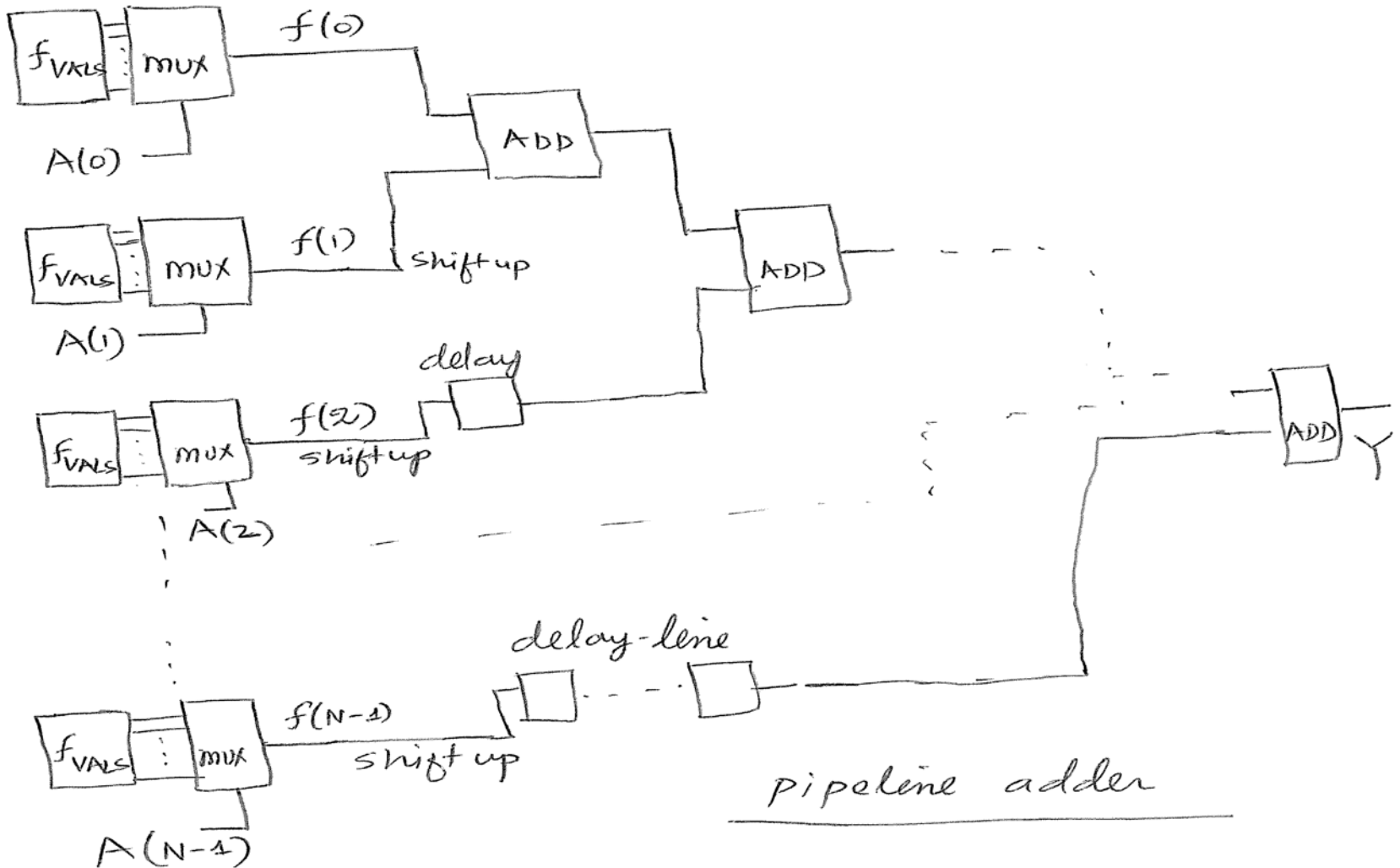
FPGA LUT IMPLEMENTATION





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FPGA LUT PIPELINE IMPLEMENTATION



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MASSEY SCIENCE SPINOFFS IN SKA RELATED Ph.D. PROJECTS

COMPLETED PhD

Stepan Lapshev

PhD Thesis topic: CMOS VLSI Implementation of Very High Speed Correlator and Beamformer Circuit for Radio Astronomical Signal Processing (*Thesis submitted, awaiting oral exam*)

Peer-reviewed journal research article:

- (1) Stepan Lapshev and S. M. Rezaul Hasan, "On the architecture for the X part of a very large FX correlator using two-accumulator CMACs," *Experimental Astronomy*, Springer, vol. 41, no. 1, pp. 259-270, 2016.
- (2) Stepan Lapshev and S. M. Rezaul Hasan, "New Low Glitch and Low Power DET Flip-Flops Using Multiple C-Elements," *IEEE Transactions On Circuits and Systems-I: Regular Papers*, vol. 63, no. 10, pp. 1673-1681, Oct. 2016.

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- (3) Stepan Lapshev and S. M. Rezaul Hasan, “Using Multiple-Accumulator CMACs to Improve Efficiency of the X Part of an Input-Buffered FX Correlator,” *Experimental Astronomy*, Springer, Vol. 43, no. 2, pp.177-187, 2017.

REPORT: SKA MEMO

- (4) Stepan Lapshev and Rezaul Hasan, *ASIC Design for Time-Domain Beamforming*, SKA CSP Memo 0023, 2017.

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IN PROGRESS:

Vignesh Raja Balu

PhD Thesis topic: Novel Architecture of Digital Array Processing Unit for Correlation in Interferometers.

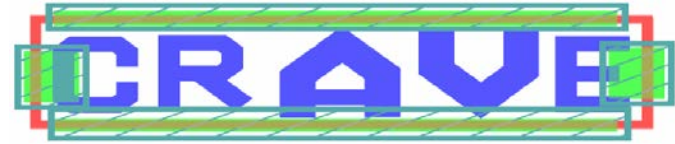
Peer-reviewed journal research article:

(5) V. R. Balu and S. M. Rezaul Hasan, "Computationally Minimized X-part for FX Correlator in Big-data Interferometers," IEEE Access, vol. 5, pp. 25353-25364, 2017.

Some more to come in 2018.....



Massey University



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Thank You!